

# Dynamics of coupled phantom and tachyon fields

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In this paper, we apply the dynamical analysis to a coupled phantom field with scaling potential taking particular forms of the coupling (linear and combination of linear), and present phase space analysis. We investigate if there exist late time accelerated scaling attractor that has the ratio of dark energy and dark matter densities of the order one. We observe that the scrutinized couplings cannot alleviate the coincidence problem, however acquire stable late time accelerated solutions. We also discuss coupled tachyon field with inverse square potential assuming linear coupling.

## I. INTRODUCTION

The late time cosmic acceleration is revealed by various observations [1–5]. A substantial efforts were put by number of authors to explore the cause of cosmic acceleration, by introducing a new player with negative pressure termed as dark energy [6]. Apart from dark energy, there are other theoretical models, such as void models and Back-reaction, which all provide late time cosmic acceleration [7].

Dark energy may be dynamical or constant. One of the simplest form of dark energy is the cosmological constant (often referred to as the  $\Lambda$ CDM model), formulated as vacuum energy that has a constant energy density and the equation of state  $w = -1$ . Although the  $\Lambda$ CDM model is supported by the present observations, it suffers two severe problems, such as coincidence and cosmological constant [8]. To circle these problems, the dynamical dark energy is a good alternative. The latter can be understood in terms of a scalar field appearing in some fundamental theories, for example, string/M-Theory. Such models have been extensively studied in the past few decades [9–21]. It has been found that the form of dark energy that dominates the present Universe could be a phantom energy, quintessence or cosmological constant. The available cosmological data do not fix a microscopic theory of dark energy. But the overall uncertainty is reflected by the existence of various phenomenological models. To reduce the number of models one way is to consider only the ones that do not violate any of the fundamental theories. The number can be further reduced by testing the models against the cosmological data.

Phantom field can be a source of dark energy and may arise from higher order theories of gravity, for example, the Brans–Dicke and non-minimally coupled scalar field theories [22, 23]. Recently, the dynamics of a coupled phantom field with dark matter has been discussed [24]. To solve the long standing cosmological constant and coincidence problems, we consider scalar fields (specifically phantom and tachyon) as a dynamical dark energy interacting with dark matter by transferring energy between the two dark components.

For an exponential potential, the quantity  $\lambda = -V'/\kappa V$ , which corresponds to the relative slope of the potential, is constant. Therefore, it is easy to study the stability of the stationary points in the phase space [25]. In the case of a tachyon field the constant  $\lambda$  corresponds to the inverse square potential.

In this letter, we investigate the stationary points and their stability for coupled phantom and tachyon fields. We apply dynamical system analysis to study the asymptotic behavior of the cosmological models mentioned above. We consider the forms of coupling that is proportional to the time derivative of their energy densities. The different forms of coupling have been studied in [26–31]. There also exist studies of the models without such particular forms of coupling [32]. The rest of the paper is organized as follows: In Sect. II we discuss the coupled phantom dynamics and construct the autonomous system which is useful for phase space analysis. In Sect. III we study phase space trajectories, and obtain stationary points and their stabilities for different forms of coupling. The stationary points and their stabilities of a tachyon field with the coupling  $Q = \beta\dot{\phi}$  is discussed in Sect. IV. We summarize our results in Sect. V.

## II. COUPLED PHANTOM DYNAMICS

In a spatially flat Universe, we consider two components, namely phantom field and matter (Baryonic+DM). The energy density of each component may not be conserved, although the total energy

density of the Universe is. Therefore, the conservation laws of energy can be written as

$$\begin{aligned}\dot{\rho}_m + 3H(\rho_m + p_m) &= Q, \\ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) &= -Q, \\ \dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) &= 0,\end{aligned}\tag{1}$$

where  $\rho_{tot} = \rho_\phi + \rho_m$  and  $p_{tot} = p_\phi + p_m$ , and  $\rho_m$ ,  $\rho_\phi$ ,  $p_m$  and  $p_\phi$  are the energy densities and pressures of matter (dust) and phantom field, respectively. The coupling is through the function  $Q$ , and  $H$  denotes the Hubble parameter.

The flow of energy between two components depends on the sign of  $Q$ . If  $Q > 0$ , the transfer of energy takes place from phantom to matter, whereas for  $Q < 0$  it occurs from matter to phantom. At the present, several forms of  $Q$  have been investigated [33–41]. Following equation (1), it is clear that  $Q$  should be a function of  $H$ ,  $\rho_m$  and  $\rho_\phi$ ,

$$Q = Q(H, \rho_m, \rho_\phi).\tag{2}$$

In this Letter, we consider three particular forms of  $Q$ :  $\alpha\dot{\rho}_m$ ,  $\beta\dot{\rho}_\phi$  and  $\sigma(\dot{\rho}_m + \dot{\rho}_\phi)$ . In these forms,  $H$  is not directly involved, as it has the dimension of the inverse of time, and the latter is already present in  $\dot{\rho}_i$ .

In a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe, the evolution equations are given by

$$\begin{aligned}H^2 &= \frac{\kappa^2}{3}(\rho_m + \rho_\phi) \\ 2\dot{H} + 3H^2 &= -\kappa^2 p_\phi\end{aligned}\tag{3}$$

where  $\kappa^2 = 8\pi G$ ,  $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$  and  $p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$ . To cast the evolution equations into an autonomous system, we introduce the following dimensionless quantities,

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}; \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}; \quad \lambda = -\frac{V'}{\kappa V}\tag{4}$$

Hence, we find

$$\begin{aligned}\frac{dx}{dN} &= x \left( \frac{\ddot{\phi}}{H\dot{\phi}} - \frac{\dot{H}}{H^2} \right) \\ \frac{dy}{dN} &= -y \left( \sqrt{\frac{3}{2}}\lambda x + \frac{\dot{H}}{H^2} \right)\end{aligned}\tag{5}$$

where,  $N = \ln a$ . For an exponential potential, we find that  $\lambda$  is constant, and

$$\frac{\dot{H}}{H^2} = \frac{3(x^2 + y^2 - 1)}{2}\tag{6}$$

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{3/2} \frac{\lambda y^2}{x} + \frac{Q}{H\dot{\phi}^2}\tag{7}$$

Then, the effective equation of state, the field density parameter and the equation of state for a phantom field are given, respectively, by

$$\begin{aligned}w_{eff} &= -1 - \frac{2\dot{H}}{3H^2} \\ \Omega_\phi &= \frac{\kappa^2 \rho_\phi}{3H^2} = -x^2 + y^2 \\ w_\phi &= \frac{w_{eff}}{\Omega_\phi}\end{aligned}\tag{8}$$

For an accelerating Universe, we have  $w_{eff} < -\frac{1}{3}$ .

### III. STATIONARY POINTS AND THEIR STABILITIES

To study stationary points and their stabilities, let us consider the autonomous system (5), from which we can find the stationary points by setting the left-hand side of these equations to zero. Then, the signs of the eigenvalues will tell us the stability of the points. In the following subsections, we consider different forms of the coupling.

#### A. Coupling $Q = \alpha \dot{\rho}_m$

For this coupling, equation (7) takes the form,

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{3/2} \frac{\lambda y^2}{x} - \frac{3\alpha\Omega_m}{2(1-\alpha)x^2} \quad (9)$$

where,  $\Omega_m = 1 - \Omega_\phi$ . Then, the autonomous system can be written as

$$\begin{aligned} \frac{dx}{dN} &= x \left( -3 - \sqrt{3/2} \frac{\lambda y^2}{x} - \frac{3\alpha\Omega_m}{2(1-\alpha)x^2} - \frac{3(x^2 + y^2 - 1)}{2} \right) \\ \frac{dy}{dN} &= -y \left( \sqrt{\frac{3}{2}}\lambda x + \frac{3(x^2 + y^2 - 1)}{2} \right) \end{aligned} \quad (10)$$

The critical points can be obtained by putting  $\frac{dx}{dN} = 0$  and  $\frac{dy}{dN} = 0$ , simultaneously. Therefore, we have the following stationary points:

- (1)  $x = -\sqrt{\frac{\alpha}{\alpha-1}}$ ,  $y = 0$ . In this case, the corresponding eigenvalues are,

$$\begin{aligned} \mu_1 &= -6 - \frac{3}{\alpha-1} < 0, & \text{for } \alpha > 1, \\ \mu_2 &= \frac{-3 + \sqrt{6\alpha(\alpha-1)}\lambda}{2(\alpha-1)} < 0, & \text{for } \alpha > 1, \sqrt{6\alpha(\alpha-1)}\lambda \leq 0 \end{aligned}$$

The point has negative eigenvalues for  $\alpha > 1$  and  $\sqrt{6\alpha(\alpha-1)}\lambda \leq 0$ . Thus, it is a stable point.

- (2)  $x = \sqrt{\frac{\alpha}{\alpha-1}}$ ,  $y = 0$ . Then, we have following eigenvalues,

$$\begin{aligned} \mu_1 &= -6 - \frac{3}{\alpha-1} < 0, & \text{for } \alpha > 1, \\ \mu_2 &= \frac{-3 - \sqrt{6\alpha(\alpha-1)}\lambda}{2(\alpha-1)} < 0, & \text{for } \alpha > 1, \sqrt{6\alpha(\alpha-1)}\lambda \geq 0 \end{aligned}$$

The eigenvalues of this point show their negativity for  $\alpha > 1$  and  $\sqrt{6\alpha(\alpha-1)}\lambda \geq 0$ . Therefore, it is a stable point.

- (3)  $x = -\frac{\lambda}{\sqrt{6}}$ ,  $y = -\sqrt{1 + \frac{\lambda^2}{6}}$ . In this case, the eigenvalues are given by,

$$\begin{aligned} \mu_1 &= -3 - \lambda^2/2 < 0, & \text{for } \lambda > 0, \\ \mu_2 &= 3/(\alpha-1) - \lambda^2 < 0, & \text{for } \alpha > 1, \lambda > \sqrt{3/(\alpha-1)} \end{aligned}$$

The point is stable under above given conditions.

- (4)  $x = -\frac{\lambda}{\sqrt{6}}$ ,  $y = \sqrt{1 + \frac{\lambda^2}{6}}$ . In this case, we get same eigenvalues as (3).

- (5)  $x = \frac{\sqrt{\frac{3}{2}}}{\lambda(1-\alpha)}$ ,  $y = -\frac{\sqrt{(\alpha-1)\alpha\lambda^2 - \frac{3}{2}}}{\lambda(\alpha-1)}$ . In this case, the corresponding eigenvalues are,

$$\begin{aligned} \mu_1 &= -\frac{1}{4} \left( 12 + \frac{9}{\alpha-1} - 2\alpha\lambda^2 + \delta_1 \right) < 0, & \text{for } 12 + \frac{9}{\alpha-1} - 2\alpha\lambda^2 + \delta_1 > 0, \\ \mu_2 &= -\frac{1}{4} \left( 12 + \frac{9}{\alpha-1} - 2\alpha\lambda^2 - \delta_1 \right) < 0, & \text{for } 12 + \frac{9}{\alpha-1} - 2\alpha\lambda^2 - \delta_1 > 0, \end{aligned}$$

where  $\delta_1 = \frac{\sqrt{(\alpha-1)\lambda^2(216 + (\alpha-1)\lambda^2(-63 + 4\alpha(-54 + 36\alpha - 3(\alpha-1)(4\alpha-5)\lambda^2 + (\alpha-1)^2\alpha\lambda^4)))}}{\lambda^2(\alpha-1)^2}$ . The point now is a saddle point.

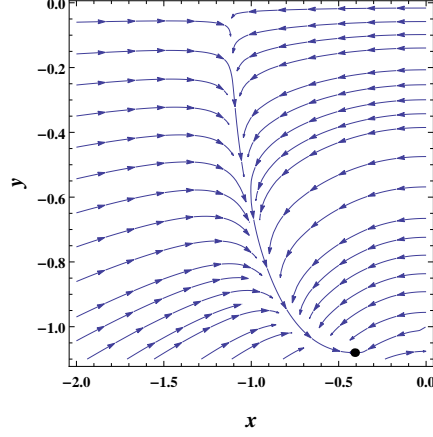


FIG. 1: The figure shows the phase space trajectories for point (3) of the coupling  $Q = \alpha \dot{\rho}_m$ . The stable fixed point is an attractive node and corresponds to  $\alpha = 5$  and  $\lambda = 1$ . The black dot represents the stable attractor point.

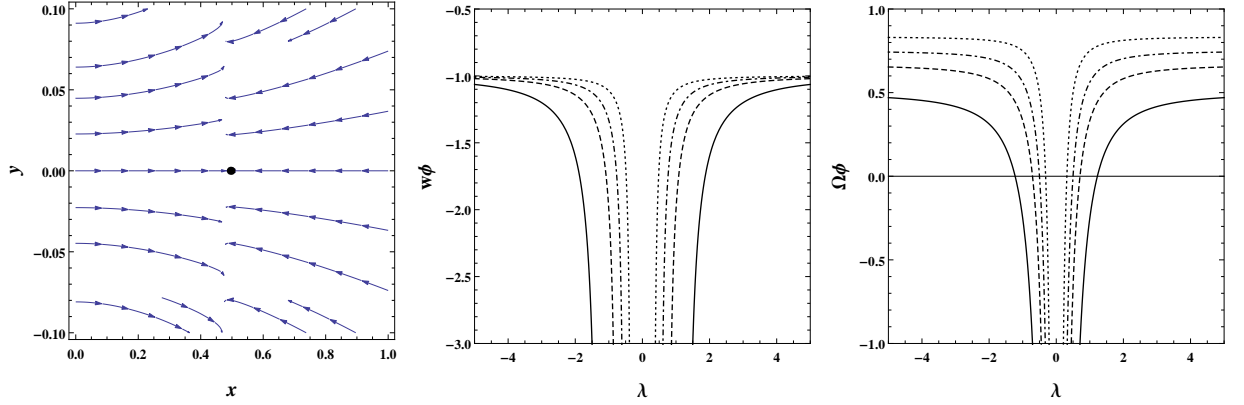


FIG. 2: This figure represents the phase portrait, evolution of  $w_\phi$  and  $\Omega_\phi$  of point (5) for  $Q = \alpha \dot{\rho}_m$ . This is an unstable point and acts as a saddle point that is shown in the left panel for  $\alpha = -0.3$  and  $\lambda = 1.9$ . The middle and right panels are plotted for different values of  $\alpha$ . The solid, dashed, dot-dashed and dotted lines correspond to  $\alpha = -1, -2, -3$  and  $-5$ , respectively. The values of  $\lambda$  below horizontal line are not allowed.

In this coupling, we are interested in Cases (3) and (5), as Case (3) is stable and has an accelerating period, whereas Case (5) is a saddle point, and also has an accelerating period. For Case (3), we solve the autonomous system (10) numerically for  $\alpha = 5$  and  $\lambda = 1$ , and the result is displayed in Fig. 1. The stable point of Case (3) acts as an attractive node under the chosen parameters which is confirmed by Fig. 1. Additionally, in this case we obtain  $\Omega_\phi = 1$  that corresponds to the case where dark energy totally dominates. However, we find that Case (3) is a stable fixed point with a late accelerating Universe ( $w_{eff} < -1/3$ ), but it can not solve the coincidence problem as it has  $\Omega_{DE} = 1$  rather than  $\Omega_{DE}/\Omega_{DM} \simeq \mathcal{O}(1)$ . In Case (5), we evolve the system (10) numerically for  $\alpha = -0.3$  and  $\lambda = 1.9$ , and find that the nature of this point is a saddle point, which is shown at the left panel of Fig. 2. We also find the cosmological observables  $\Omega_\phi$ ,  $w_{eff}$  and  $w_\phi$ . The middle and right panels of Fig. 2 show the evolution of  $w_\phi$  and  $\Omega_\phi$  versus  $\lambda$ . They also show for which range of  $\lambda$  (having different values of  $\alpha$ ) both physical observables are allowed. The general properties of this coupling are summarized in Table I.

### B. Coupling $Q = \beta \dot{\rho}_\phi$

For the coupling  $Q = \beta \dot{\rho}_\phi$ , equation (7) becomes,

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{3/2} \frac{\lambda y^2}{x} + \frac{3\beta}{1+\beta} \quad (11)$$

TABLE I: We display stationary points for the coupling  $Q = \alpha \rho_m$ . We also show the expressions of  $\Omega_\phi$ ,  $w_{eff}$ ,  $w_\phi$  and the conditions to have an accelerating phase.

| Point | $x$                                    | $y$   | Stability   | $\Omega_\phi$   | $w_{eff}$                  | $w_\phi = \frac{w_{eff}}{\Omega_\phi}$                           | Acceleration |
|-------|--|---|---|---|----------------------------|--|--------------|
| 1     | $-\sqrt{\frac{\alpha}{\alpha-1}}$      | 0   | Stable for $\alpha > 1$ ,<br>$\sqrt{6\alpha(\alpha-1)}\lambda \leq 0$ | $\frac{\alpha}{1-\alpha}$                                   | $\frac{\alpha}{1-\alpha}$  | 1  | No           |
| 2     | $\sqrt{\frac{\alpha}{\alpha-1}}$       | 0   | Stable for $\alpha > 1$ ,<br>$\sqrt{6\alpha(\alpha-1)}\lambda \geq 0$ | $\frac{\alpha}{1-\alpha}$                                   | $\frac{\alpha}{1-\alpha}$  | 1  | No           |
| 3,4   | $-\frac{\lambda}{\sqrt{6}}$            | $\mp\sqrt{1 + \frac{\lambda^2}{6}}$                               | Stable for $\alpha > 1$ ,<br>$\lambda > \sqrt{\frac{3}{\alpha-1}}$    | 1   | $-1 - \frac{\lambda^2}{3}$ | $-1 - \frac{\lambda^2}{3}$                                       | Yes          |
| 5     | $\frac{\sqrt{3/2}}{\lambda(1-\alpha)}$ | $-\frac{\sqrt{(\alpha-1)\alpha\lambda^2-3/2}}{\lambda(\alpha-1)}$ | Saddle for<br>$12 + \frac{9}{\alpha-1} - 2\alpha\lambda^2 > \delta_1$ | $\frac{(\alpha-1)\alpha\lambda^2-3}{(\alpha-1)^2\lambda^2}$ | $\frac{\alpha}{1-\alpha}$  | $-\frac{(\alpha-1)\alpha\lambda^2}{(\alpha-1)\alpha\lambda^2-3}$ | Yes          |

Therefore, equation (5) takes the form,

$$\begin{aligned} \frac{dx}{dN} &= x \left( -3 - \sqrt{3/2} \frac{\lambda y^2}{x} + \frac{3\beta}{1+\beta} - \frac{3(x^2 + y^2 - 1)}{2} \right) \\ \frac{dy}{dN} &= -y \left( \sqrt{\frac{3}{2}}\lambda x + \frac{3(x^2 + y^2 - 1)}{2} \right) \end{aligned} \quad (12)$$

For this coupling, we have the following stationary points:

- (1)  $x = 0$ ,  $y = 0$ . In this case, the corresponding eigenvalues are,

$$\begin{aligned} \mu_1 &= \frac{3}{2} - \frac{3}{1+\beta} < 0, & \text{for } 0 < \beta < 1, \\ \mu_2 &= \frac{3}{2} \end{aligned}$$

As one of the eigenvalue is positive, the stationary point is unstable for any value of  $\beta$ .

- (2)  $x = -\sqrt{\frac{\beta-1}{\beta+1}}$ ,  $y = 0$ . In this case, the eigenvalues are given as,

$$\begin{aligned} \mu_1 &= -3 + \frac{6}{1+\beta} < 0, & \text{for } \beta < -1, \\ \mu_2 &= \frac{3}{1+\beta} + \sqrt{\frac{3}{2} - \frac{3}{1+\beta}}\lambda < 0, & \text{for } -2 \leq \beta < -1 \text{ and } 0 < \lambda \leq 1, \end{aligned}$$

The eigenvalues of this point show the negativity for  $-2 \leq \beta < -1$  and  $0 < \lambda \leq 1$ . Therefore, it is a stable point.

(3)  $x = \sqrt{\frac{\beta-1}{\beta+1}}$ ,  $y = 0$ . In this case, the corresponding eigenvalues are,

$$\begin{aligned}\mu_1 &= -3 + \frac{6}{1+\beta} < 0, & \text{for } \beta < -1, \\ \mu_2 &= \frac{3}{1+\beta} - \sqrt{\frac{3}{2} - \frac{3}{1+\beta}}\lambda < 0, & \text{for } \beta < -1 \text{ and } \lambda > 0,\end{aligned}$$

It is stable point for above given conditions.

(4)  $x = \frac{9-(1+\beta)^2\lambda^4+\delta_2}{2\sqrt{6}\lambda(1+\beta)(3+(1+\beta)\lambda^2)}$ ,  $y = -\frac{\sqrt{6(1+\beta)^2\lambda^2-9+\lambda^4(1+\beta)^2-\delta_2}}{2\sqrt{3}(1+\beta)\lambda}$ . In this case, we have following eigenvalues,

$$\begin{aligned}\mu_1 &= -\frac{2\delta_2^2 - 6(1+\beta)\delta_2\epsilon\lambda^2 + 2\epsilon^2(-9 + (1+\beta)\lambda^2(9 + 2(1+\beta)\lambda^2)) + \nu}{16(1+\beta)^2\epsilon^2\lambda^2} < 0, \\ &\text{for } 2\delta_2^2 + 2\epsilon^2(-9 + (1+\beta)\lambda^2(9 + 2(1+\beta)\lambda^2)) + \nu < 0, \\ \mu_2 &= -\frac{2\delta_2^2 - 6(1+\beta)\delta_2\epsilon\lambda^2 + 2\epsilon^2(-9 + (1+\beta)\lambda^2(9 + 2(1+\beta)\lambda^2)) - \nu}{16(1+\beta)^2\epsilon^2\lambda^2} < 0, \\ &\text{for } 2\delta_2^2 + 2\epsilon^2(-9 + (1+\beta)\lambda^2(9 + 2(1+\beta)\lambda^2)) - \nu < 0,\end{aligned}$$

where,

$$\begin{aligned}\delta_2 &= \sqrt{(3 + (1+\beta)\lambda^2)^2(9 + (1+\beta)\lambda^2(6 + \lambda^2 + \beta(12 + \lambda^2)))} \\ \epsilon &= 3 + (1+\beta)\lambda^2 \\ \nu &= \sqrt{(\delta_2^4 - 12(1+\beta)\lambda^2\delta_2^3\epsilon - 6\delta_2^2\epsilon^2(3 - 4(\beta-6)(1+\beta)\lambda^2 + 3(1+\beta)^2\lambda^4) + \epsilon^4(-9 + 12(1+\beta)(2+\beta)\lambda^2 \\ &\quad + 5(1+\beta)^2\lambda^4)^2 + 4(1+\beta)\lambda^2\delta_2\epsilon^3(-117 + (1+\beta)\lambda^2(24 + \lambda^2 + \beta(60 + \lambda^2))))}\end{aligned}$$

The eigenvalues of this point show the negativity under above conditions. Hence, it is a stable point. The  $\Omega_\phi$ ,  $w_{eff}$  and  $w_\phi$  are given as,

$$\Omega_\phi = -\frac{9 - (1+\beta)^2\lambda^4 + \sqrt{\delta_2}}{2(1+\beta)^2\lambda^2\epsilon} \quad (13)$$

$$w_{eff} = -\frac{9 + 18\beta + 6(1+\beta)^2\lambda^2 + (1+\beta)^2\lambda^4 - \sqrt{\delta_2}}{6(1+\beta)\epsilon} \quad (14)$$

$$w_\phi = \frac{-9 - (1+\beta)\lambda^2(6 + (1+\beta)\lambda^2) + \sqrt{\gamma}}{6\epsilon} \quad (15)$$

where,

$$\gamma = 81 + (1+\beta)^2\lambda^2(108 + 18(3+4\beta)\lambda^2 + 12(1+\beta)^2\lambda^4 + (1+\beta)^2\lambda^6) \quad (16)$$

For this coupling, we pay particular attention on Cases (2) and (4). In case (2), we evolve the autonomous system (12) numerically for the values  $\beta = -2$  and  $\lambda = 1$ , and get  $\Omega_\phi$ ,  $w_{eff}$  and  $w_\phi$ . With the chosen parameters, the point is stable and behaves as an attractive node (see Fig. 3), but there does not exist an accelerating phase of the Universe, as the equation of state  $w_\phi$  for phantom field is always positive. Therefore, it does not solve the coincidence problem. In Case (4), we elaborate the system for  $\beta = -2.5$  and  $\lambda = 1$ , and find that it is stable and acts as an attractive node. The phase portrait of this stable point is shown in the left panel of Fig. 4, the middle and right panels of Fig. 4 show the evolution of  $w_\phi$  and  $\Omega_\phi$ . For this point, we consider two cases: (a)  $\beta = -2.5$  and  $1 \leq \lambda < 1.5$ , in which case (4) behaves as a stable point but does not give rise to an accelerating Universe as  $w_\phi$  is always positive. (b)  $\beta = -2.5$  and  $\lambda > 1.5$ , in which Case (4) acts as a saddle point and has an accelerating Universe phase as  $w_\phi < -1$  (see Table II). Hence, it does not alleviate the coincidence problem. The results of the coupling are summarized in Table II.

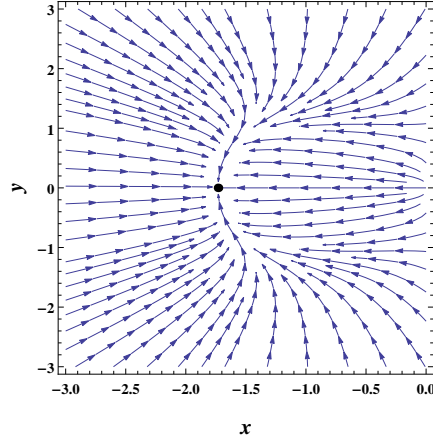


FIG. 3: The figure displays the phase space trajectories of Case (2) for  $Q = \beta \dot{\rho}_\phi$ . It is plotted for  $\beta = -2$  and  $\lambda = 1$ . The point is stable and behaves as an attractive node.

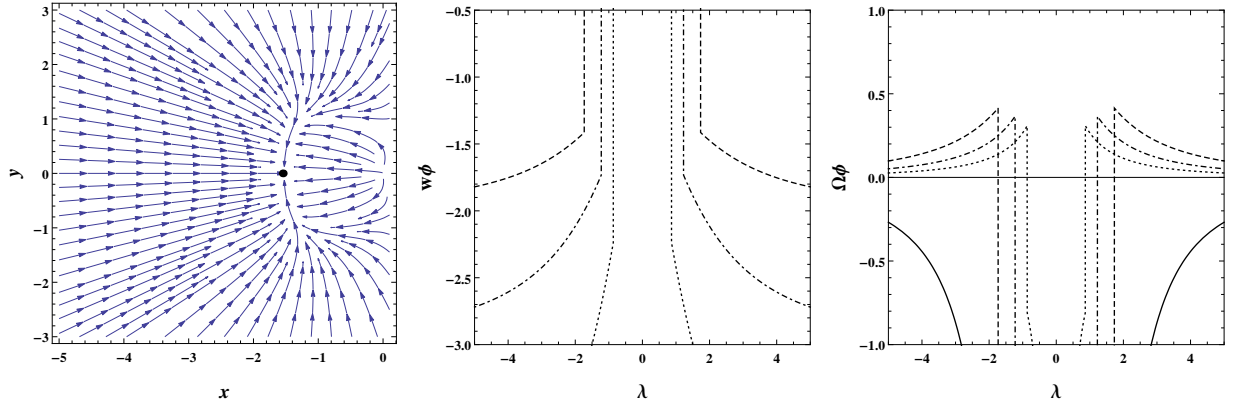


FIG. 4: The left panel shows the phase portrait of Case (4) for  $Q = \beta \dot{\rho}_\phi$ , and corresponds to  $\beta = -2.5$  and  $\lambda = 1$ . The middle and right panels show the evolution of  $w_\phi$  and  $\Omega_\phi$  versus  $\lambda$  for various values of  $\beta$ . The solid, dashed, dot-dashed and dotted lines correspond to  $\beta = -0.5, -2, -3$  and  $-5$ , respectively. The values of  $\lambda$  below the horizontal line are not accepted. This is a stable point and acts as an attractive node.

### C. Coupling $Q = \sigma(\dot{\rho}_m + \dot{\rho}_\phi)$

In this case, the coupling  $Q$  is a linear combination of  $\dot{\rho}_m$  and  $\dot{\rho}_\phi$ . For this coupling, equation (7) can be written as,

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{3/2} \frac{\lambda y^2}{x} - \frac{3\sigma\Omega_m}{2(1-\sigma)x^2} + \frac{3\sigma}{1+\sigma}, \quad (17)$$

Thus, the autonomous system (5) becomes,

$$\begin{aligned} \frac{dx}{dN} &= x \left( -3 - \sqrt{3/2} \frac{\lambda y^2}{x} - \frac{3\sigma\Omega_m}{2(1-\sigma)x^2} + \frac{3\sigma}{1+\sigma} - \frac{3(x^2 + y^2 - 1)}{2} \right) \\ \frac{dy}{dN} &= -y \left( \sqrt{\frac{3}{2}} \lambda x + \frac{3(x^2 + y^2 - 1)}{2} \right) \end{aligned} \quad (18)$$

For this coupling, we have the following stationary points:

TABLE II: We present stationary points and their stability for the coupling  $Q = \beta\dot{\rho}_\phi$ .

| Point | $x$  | $y$ | Stability   | $\Omega_\phi$             | $w_{eff}$                 | $w_\phi = \frac{w_{eff}}{\Omega_\phi}$ | Acceleration |
|-------|--|-----|---|---------------------------|---------------------------|--|--------------|
| 1     | 0  | 0   | Saddle  | 0                         | 0                         | Indeterminate                          | No           |
| 2     | $-\sqrt{\frac{\beta-1}{\beta+1}}$  | 0   | Stable for $-2 \leq \beta < -1$<br>and $0 < \lambda \leq 1$ | $\frac{1-\beta}{1+\beta}$ | $\frac{1-\beta}{1+\beta}$ | 1                                      | No           |
| 3     | $\sqrt{\frac{\beta-1}{\beta+1}}$   | 0   | Stable for $\beta < -1$<br>and $\lambda > 0$                | $\frac{1-\beta}{1+\beta}$ | $\frac{1-\beta}{1+\beta}$ | 1                                      | No           |
| 4     | $\frac{9-(1+\beta)^2\lambda^4+\delta_2}{2\sqrt{6\lambda(1+\beta)(3+(1+\beta)\lambda^2)}} - \frac{\sqrt{6(1+\beta)^2\lambda^2-9+\lambda^4(1+\beta)^2-\delta_2}}{2\sqrt{3(1+\beta)\lambda}}$ |     | —   | Eq.(13)                   | Eq.(14)                   | Eq.(15)                                | —            |
|       |  |     | (a) Stable for $\beta = -2.5$<br>and $1 \leq \lambda < 1.5$ |                           |                           | Positive                               | No           |
|       |  |     | (b) Saddle for $\beta = -2.5$<br>and $\lambda \geq 1.5$     |                           |                           | $< -1$                                 | Yes          |

(1)  $x = -\frac{\sqrt{1-\sigma+2\sigma^2}-\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sqrt{2(\sigma^2-1)}}$ ,  $y = 0$ . In this case, the corresponding eigenvalues are,

$$\mu_1 = \frac{3\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sigma^2-1} < 0, \text{ for } \sigma^2 < 1,$$

$$\mu_2 = \frac{1}{8} \left( \frac{6(\sigma-3+\sqrt{1+\sigma(\sigma+8\sigma^3-6)})}{\sigma^2-1} + 4\sqrt{3}\lambda\sqrt{\frac{1-\sigma+2\sigma^2-\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sigma^2-1}} \right) > 0, \text{ for all } \sigma.$$

As one of the eigenvalue is positive, the stationary point is a saddle for any value of  $\sigma$ .

(2)  $x = \frac{\sqrt{1-\sigma+2\sigma^2}-\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sqrt{2(\sigma^2-1)}}$ ,  $y = 0$ . In this case, the eigenvalues are given as,

$$\mu_1 = \frac{3\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sigma^2-1} < 0, \text{ for } \sigma^2 < 1,$$

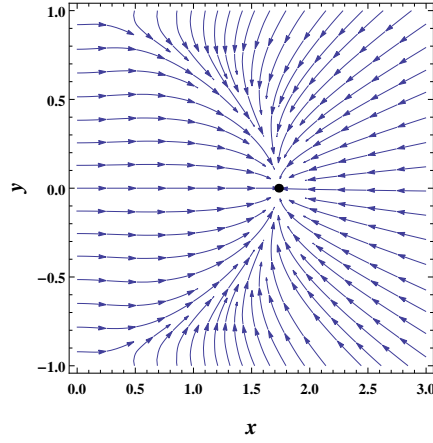
$$\mu_2 = \frac{1}{8} \left( \frac{6(\sigma-3+\sqrt{1+\sigma(\sigma+8\sigma^3-6)})}{\sigma^2-1} - 4\sqrt{3}\lambda\sqrt{\frac{1-\sigma+2\sigma^2-\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sigma^2-1}} \right) > 0, \text{ for all } \sigma.$$

This is a saddle point.



TABLE III: We show stationary points for the coupling  $Q = \sigma(\dot{\rho}_m + \dot{\rho}_\phi)$ .

| Point | $x$  | $y$ | Stability                                 | $\Omega_\phi$   | $w_{eff}$   | $w_\phi = \frac{w_{eff}}{\Omega_\phi}$ | Acceleration |
|-------|--|-----|---|---|---|--|--------------|
| 1, 2  | $\mp \frac{\sqrt{1-\sigma+2\sigma^2}-\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sqrt{2(\sigma^2-1)}}$ | 0   | Saddle                                    | $\frac{\sigma-1-2\sigma^2}{2(\sigma^2-1)}$<br>$+\frac{\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{2(\sigma^2-1)}$  | $\frac{\sigma-1-2\sigma^2}{2(\sigma^2-1)}$<br>$+\frac{\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{2(\sigma^2-1)}$  | 1                                      | No           |
| 3     | $\sqrt{\frac{1+\sigma(2\sigma-1)+\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{2(\sigma^2-1)}}$           | 0   | Stable for<br>$\sigma^2 > 1, \lambda > 0$ | $\frac{1+\sigma(2\sigma-1)}{2(1-\sigma^2)}$<br>$+\frac{\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{2(1-\sigma^2)}$ | $\frac{1+\sigma(2\sigma-1)}{2(1-\sigma^2)}$<br>$+\frac{\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{2(1-\sigma^2)}$ | 1                                      | No           |

FIG. 5: The figure represents the evolution of the phase space trajectories of Case (3) for  $Q = \sigma(\dot{\rho}_m + \dot{\rho}_\phi)$ , and is plotted for  $\sigma = 2$  and  $\lambda = 1$ . The stable point acts as an attractive node, and the black dot designates a stable attractor point.

(3)  $x = \sqrt{\frac{1+\sigma(2\sigma-1)+\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{2(\sigma^2-1)}}$ ,  $y = 0$ . In this case, the eigenvalues take the form,

$$\mu_1 = -\frac{3\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sigma^2-1} < 0, \text{ for } \sigma^2 > 1,$$

$$\mu_2 = \frac{1}{8} \left( -\frac{6(3-\sigma+\sqrt{1+\sigma(\sigma+8\sigma^3-6)})}{\sigma^2-1} - 4\sqrt{3}\lambda \sqrt{\frac{1+\sigma(2\sigma-1)+\sqrt{1+\sigma(\sigma+8\sigma^3-6)}}{\sigma^2-1}} \right) < 0,$$

for  $\sigma^2 > 1$  and  $\lambda > 0$ .

The eigenvalues of the point show the negativity for  $\sigma^2 > 1$  and  $\lambda > 0$ . Therefore, it is a stable point.

For this coupling, the stationary point in Case (3) is stable for  $\sigma^2 > 1$  and  $\lambda > 0$ . We numerically evolve the autonomous system (18) for the choices  $\sigma = 2$  and  $\lambda = 1$ . The phase space trajectories of the stable point is displayed in Fig. 5, and the point behaves as an attractive node. For this point we do not find any accelerating solution as it has positive equation of state. Hence, it can not solve the coincidence problem. The main results of this coupling are summarized in Table III.

#### IV. COUPLED TACHYON DYNAMICS

Tachyon acts as dark energy, depending on the shape of the potential [42]. We consider that dark energy and dark matter are interacting to each other, but the total energy density is conserved. The conservation equations for both components are written as,

$$\begin{aligned}\dot{\rho}_m + 3H(\rho_m + p_m) &= Q, \\ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) &= -Q,\end{aligned}\tag{19}$$

where,

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}\tag{20}$$

Then, the evolution equations take the form,

$$H^2 = \frac{\kappa^2}{3} \left[ \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + \rho_m \right],\tag{21}$$

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{V'(\phi)}{V(\phi)} = -\frac{Q\sqrt{1 - \dot{\phi}^2}}{\dot{\phi}V(\phi)}\tag{22}$$

where a prime and a dot denote derivative with respect to field and cosmic time, respectively.

Let us define the following dimensionless parameters

$$x = \dot{\phi}, \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \quad \Omega_m = \frac{\kappa^2\rho_m}{3H^2}, \quad \lambda = -\frac{V'}{\kappa V\sqrt{V}}\tag{23}$$

Then, we obtain the autonomous system,

$$\begin{aligned}\frac{dx}{dN} &= \frac{\ddot{\phi}}{H\dot{\phi}} x \\ \frac{dy}{dN} &= -\frac{\sqrt{3}}{2}y^2\lambda x - y \left( \frac{\dot{H}}{H^2} \right)\end{aligned}\tag{24}$$

Here we take inverse square potential for which  $\lambda$  is constant. Also, we consider the coupling  $Q = \beta\dot{\rho}_\phi$  only. For this coupling we have following equations,

$$\frac{\dot{H}}{H^2} = \frac{3(y^2\sqrt{1 - x^2} - 1)}{2},\tag{25}$$

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3(1 - x^2) + \sqrt{3}\lambda y \frac{(1 - x^2)}{x} + \frac{3\beta(1 - x^2)}{1 + \beta},\tag{26}$$

The equation of state for the tachyon field is given as,

$$w_{eff} = -1 - \frac{2\dot{H}}{3H^2},\tag{27}$$

$$w_\phi = \frac{w_{eff} - w_m\Omega_m}{1 - \Omega_m},\tag{28}$$

where  $w_m = 0$  for standard dust matter. Setting the left hand sides of the autonomous system (24) to zero, we obtain the following stationary points:

(1)  $x = 0, y = 0$ . In this case, the corresponding eigenvalues are,

$$\begin{aligned}\mu_1 &= -\frac{3}{1 + \beta} < 0, \quad \text{for } 0 < \beta < 1, \\ \mu_2 &= \frac{3}{2},\end{aligned}$$

As one of the eigenvalue is positive, the stationary point is a saddle.

(2)  $x = \pm 1$ ,  $y = \pm \frac{\sqrt{3}}{\lambda}$ . In this case, the metric is in-determinant.

$$(3) \quad x = -\frac{1}{3\sqrt{6}}(1+\beta)\lambda\sqrt{\left(\left((-81(2 \times 2^{1/3}\delta_3^{2/3} + 18(18 + \delta_5)) + 2^{2/3}\delta_3^{1/3}(18 + \delta_5)) + 9(1+\beta)^2(243\beta^2(18 + 2^{2/3}\delta_4^{1/3}) + 12\beta(486 + 27 \times 2^{2/3}\delta_4^{1/3} - 2^{1/3}\delta_4^{2/3}) + 135 \times 2^{2/3}\delta_4^{1/3} - 8 \times 2^{1/3}\delta_4^{2/3} + 18(153 + \delta_5))\lambda^4 - 2(1+\beta)^4(243 + 2187\beta^2 - 81\beta(-18 + 2^{2/3}\delta_4^{1/3}) - 54 \times 2^{2/3}\delta_4^{1/3} + 2^{1/3}\delta_4^{2/3})\lambda^8 + 2(1+\beta)^6(-90 - 162\beta + 2^{2/3}\delta_4^{1/3})\lambda^{12} - 4(1+\beta)^8\lambda^{16}\right. \\ \left. / ((1+\beta)^2\lambda^2(-81(18 + \delta_5) + 243(1+\beta)^2(5 + 3\beta(4 + 3\beta)))\lambda^4 + 54(1+\beta)^4(2 + 3\beta)\lambda^8 + 2(1+\beta)^6\lambda^{12}))\right)}$$

$$y = -\frac{1}{3\sqrt{2}}\sqrt{\left(\left((-81(324 + 18 \times 2^{2/3}\delta_3^{1/3} + 2 \times 2^{1/3}\delta_3^{2/3} + 18\delta_5 + 2^{2/3}\delta_3^{1/3}\delta_5) + 9(1+\beta)^2(2754 + 243\beta^2(18 + 2^{2/3}\delta_4^{1/3}) + 12\beta(486 + 27 \times 2^{2/3}\delta_4^{1/3} - 2^{1/3}\delta_4^{2/3}) + 135 \times 2^{2/3}\delta_4^{1/3} - 8 \times 2^{1/3}\delta_4^{2/3} + 18\delta_5)\lambda^4 - 2(1+\beta)^4(243 + 2187\beta^2 - 81\beta(-18 + 2^{2/3}\delta_4^{1/3}) - 54 \times 2^{2/3}\delta_4^{1/3} + 2^{1/3}\delta_4^{2/3})\lambda^8 + 2(1+\beta)^6(-90 - 162\beta + 2^{2/3}\delta_4^{1/3})\lambda^{12} - 4(1+\beta)^8\lambda^{16}\right) \\ / ((1+\beta)^2\lambda^2(-81(18 + \delta_5) + 243(1+\beta)^2(5 + 3\beta(4 + 3\beta)))\lambda^4 + 54(1+\beta)^4(2 + 3\beta)\lambda^8 + 2(1+\beta)^6\lambda^{12}))\right)}$$

For this point, we get following eigenvalues,

$$\mu_1 = \frac{-\frac{\eta_2}{2} + 3\beta\left(\frac{-\eta_2}{6} + \sqrt{\eta_6}\right) + \frac{\eta_4\left(\frac{\eta_3}{18} + 2\sqrt{\eta_6}\right)}{6\eta_5} - 3\sqrt{\eta_6} - \eta_8 - \frac{1}{3}(1+\beta)^2\sqrt{\eta_2}\sqrt{\eta_6}\sqrt{\eta_7}\lambda^2}{4(1+\beta)\sqrt{\eta_6}} < 0$$

$$\text{for } \beta \neq -1 \text{ and } 3\beta\left(\frac{-\eta_2}{6} + \sqrt{\eta_6}\right) + \frac{\eta_4\left(\frac{\eta_3}{18} + 2\sqrt{\eta_6}\right)}{6\eta_5} < 0$$

$$\mu_2 = \frac{\frac{\eta_{10}}{2} + 3\beta\left(\frac{\eta_{10}}{6} + \sqrt{\eta_6}\right) + \frac{\eta_4\left(\frac{\eta_3}{18} + 2\sqrt{\eta_6}\right)}{6\eta_5} - 3\sqrt{\eta_6} + \eta_8 - \frac{(1+\beta)^2\sqrt{\eta_2}\sqrt{3 - \frac{\eta_4}{18\eta_5}}\sqrt{\eta_7}\lambda^2}{3\sqrt{3}}}{4(1+\beta)\sqrt{\eta_6}} < 0$$

$$\text{for } \beta \neq -1 \text{ and } \frac{\eta_{10}}{2} + 3\beta\left(\frac{\eta_{10}}{6} + \sqrt{\eta_6}\right) + \frac{\eta_4\left(\frac{\eta_3}{18} + 2\sqrt{\eta_6}\right)}{6\eta_5} + \eta_8 < 0$$

where,

$$\delta_3 = -243(1+\beta)^2(5 + 3\beta(4 + 3\beta))\lambda^4 - 54(1+\beta)^4(2 + 3\beta)\lambda^8 - 2(1+\beta)^6\lambda^{12} + 81(18 + \delta_5)$$

$$\delta_4 = 1458 - 243(1+\beta)^2(5 + 3\beta(4 + 3\beta))\lambda^4 - 54(1+\beta)^4(2 + 3\beta)\lambda^8 - 2(1+\beta)^6\lambda^{12} + 81\delta_5$$

$$\delta_5 = \sqrt{3(1+\beta)^4\lambda^4(-324 + 9(1+\beta)^2(-1 + 9\beta(2 + 3\beta))\lambda^4 + 4\beta(1+\beta)^4\lambda^8)}$$

$$\eta_1 = (-81(324 + 18 \times 2^{2/3}\delta_3^{1/3} + 2 \times 2^{1/3}\delta_3^{2/3} + 18\delta_5 + 2^{2/3}\delta_3^{1/3}\delta_5) + 9(1+\beta)^2(2754 + 243\beta^2(18 + 2^{2/3}\delta_4^{1/3}) + 12\beta(486 + 27 \times 2^{2/3}\delta_4^{1/3} - 2^{1/3}\delta_4^{2/3}) + 135 \times 2^{2/3}\delta_4^{1/3} - 8 \times 2^{1/3}\delta_4^{2/3} + 18\delta_5)\lambda^4 - 2(1+\beta)^4(243 + 2187\beta^2 - 81\beta(-18 + 2^{2/3}\delta_4^{1/3}) - 54 \times 2^{2/3}\delta_4^{1/3} + 2^{1/3}\delta_4^{2/3})\lambda^8 + 2(1+\beta)^6(-90 - 162\beta + 2^{2/3}\delta_4^{1/3})\lambda^{12} - 4(1+\beta)^8\lambda^{16}$$

$$\eta_2 = \frac{\eta_1}{(1+\beta)^2\lambda^2(-81(18 + \delta_5) + 243(1+\beta)^2(5 + 3\beta(4 + 3\beta))\lambda^4 + 54(1+\beta)^4(2 + 3\beta)\lambda^8 + 2(1+\beta)^6\lambda^{12})}$$

$$\eta_3 = \frac{\eta_1}{(1+\beta)\lambda^2(-81(18+\delta_5) + 243(1+\beta)^2(5+3\beta(4+3\beta))\lambda^4 + 54(1+\beta)^4(2+3\beta)\lambda^8 + 2(1+\beta)^6\lambda^{12})}$$

$$\begin{aligned} \eta_4 = & (-81(2 \times 2^{1/3}\delta_3^{2/3} + 18(18+\delta_5) + 2^{2/3}\delta_3^{1/3}(18+\delta_5)) + 9(1+\beta)^2(243\beta^2(18+2^{2/3}\delta_4^{1/3}) + \\ & 12\beta(486+27 \times 2^{2/3}\delta_4^{1/3} - 2^{1/3}\delta_4^{2/3}) + 135 \times 2^{2/3}\delta_4^{1/3} - 8 \times 2^{1/3}\delta_4^{2/3} + 18(153+\delta_5))\lambda^4 - 2(1+\beta)^4 \\ & (243+2187\beta^2-81\beta(-18+2^{2/3}\delta_4^{1/3})-54 \times 2^{2/3}\delta_4^{1/3}+2^{1/3}\delta_4^{2/3}\lambda^8+2(1+\beta)^6(-90-162\beta+2^{2/3}\delta_4^{1/3})\lambda^{12} \\ & - 4(1+\beta)^8\lambda^{16}) \end{aligned}$$

$$\eta_5 = -81(18+\delta_5) + 243(1+\beta)^2(5+3\beta(4+3\beta))\lambda^4 + 54(1+\beta)^4(2+3\beta)\lambda^8 + 2(1+\beta)^6\lambda^{12}$$

$$\eta_6 = 1 - \frac{\eta_4}{54\eta_5}$$

$$\eta_7 = \frac{\eta_4}{(1+\beta)^2\eta_5\lambda^2}$$

$$\begin{aligned} \eta_8 = & \sqrt{3}\sqrt{\left(\eta_6\left(3(3+\beta)^2 + \frac{\eta_3^2}{12} + \frac{\eta_4^2}{27\eta_5^2} - \frac{2}{9}(1+\beta)^2\sqrt{\eta_2}\left(-3-\beta + \frac{\eta_3\sqrt{\eta_6}}{18}\right)\sqrt{\eta_7}\lambda^2 - \frac{2}{81}(1+\beta)^4\sqrt{\eta_2}\eta_7^{3/2}\lambda^4\right.\right.} \\ & \left.\left. - \frac{1}{9}\eta_3(27\sqrt{\eta_6} + 9\beta\sqrt{\eta_6} + 4(1+\beta)\lambda^2 + \frac{\eta_4(-12(3+\beta) + \frac{1}{18}\eta_3(36\sqrt{\eta_6} - \frac{\eta_1}{2(1+\beta)\eta_5\lambda^2} + 4(1+\beta)\lambda^2))}{18\eta_5}\right)\right)} \end{aligned}$$

$$\begin{aligned} \eta_9 = & 81(324+18 \times 2^{2/3}\delta_3^{1/3} + 2 \times 2^{1/3}\delta_3^{2/3} + 18\delta_5 + 2^{2/3}\delta_3^{1/3}\delta_5) - 9(1+\beta)^2(2754+243\beta^2(18+2^{2/3}\delta_4^{1/3}) + \\ & 12\beta(486+27 \times 2^{2/3}\delta_4^{1/3} - 2^{1/3}\delta_4^{2/3}) + 135 \times 2^{2/3}\delta_4^{1/3} - 8 \times 2^{1/3}\delta_4^{2/3} + 18\delta_5)\lambda^4 + 2(1+\beta)^4(243+2187\beta^2 - \\ & 81\beta(-18+2^{2/3}\delta_4^{1/3})-54 \times 2^{2/3}\delta_4^{1/3}+2^{1/3}\delta_4^{2/3}\lambda^8-2(1+\beta)^6(-90-162\beta+2^{2/3}\delta_4^{1/3})\lambda^{12}+4(1+\beta)^8\lambda^{16}) \end{aligned}$$

$$\eta_{10} = \frac{\eta_9}{(1+\beta)^2\eta_5\lambda^2}$$

In the case of a tachyon field, we consider only the coupling  $Q = \beta\dot{\rho}_\phi$ . We are interested in Case (3) as the eigenvalues of the point are negative.

## V. CONCLUSIONS

We investigated the interaction of a phantom field with a dark matter component in a spatially flat FLRW Universe. The choices of the coupling  $Q$  in the conservation equations were phenomenological and heuristic as there is no fundamental theory of coupling strength in the dark sector was involved. We examined three different couplings, and studied the corresponding dynamical behavior and phase space. We paid attention on the stable point which could give rise to an accelerating phase. For all the three different couplings, we found  $\Omega_\phi$ ,  $w_{eff}$  and  $w_\phi$ . Our primary goal was to see if there exist late time scaling attractor with an accelerating phase and having the property  $\Omega_{DE}/\Omega_{DM} \simeq \mathcal{O}(1)$ . For the coupling  $Q = \alpha\dot{\rho}_m$ , we focused on Cases (3) and (5). In both cases the stationary points have an accelerating phase, but one of the stationary point is stable and the other is a saddle point. In case (3) the point is stable for  $\alpha > 1$  and  $\lambda > \sqrt{3/(\alpha-1)}$ , and behaves as an attractive node. In this case, we obtained a stable fixed point with an accelerating Universe ( $w_{eff} < -1/3$ ), however it corresponds to the case where dark energy completely dominates, as now we have  $\Omega_\phi = 1$ . Therefore, it does not solve the coincidence problem. The results are shown in Fig. 1 and Fig. 2. In the case of the coupling  $Q = \beta\dot{\rho}_\phi$ , we concentrated on Cases (2) and (4), and in both cases the points are stable but possessing non-accelerating phases as now  $w_\phi$  is always positive (see Table II). In the case (4), we considered two sets of the parameters as  $\beta = -2.5$ ,  $1 \leq \lambda < 1.5$  and  $\beta = -2.5$ ,  $\lambda > 1.5$ . In the first set, the point in Case (4) behaves as a stable point and give rise to a non-accelerating Universe ( $w_\phi$  always positive). In the second set, it acts as a saddle point and has an accelerating Universe ( $w_\phi < -1$ ). Thus, it can not solve coincidence problem either. The phase portrait, evolution of  $w_\phi$  and  $\Omega_\phi$  are displayed in Figs. 3

and 4. For the linear combination of the coupling  $Q = \sigma(\dot{\rho}_m + \dot{\rho}_\phi)$ , we noticed that the stationary point in Case (3) is stable for  $\sigma^2 > 1$  and  $\lambda > 0$ , but could not give rise to an accelerating Universe as the equation of state is always positive. The phase portrait for this case is shown in Fig. 5, and it acts as an attractive node.

For all the couplings considered here, our analysis showed that the coincidence problem cannot be alleviated in the coupled phantom field models. Similar results were also shown in [43] for different couplings.

We also studied the dynamical behavior and stabilities for the coupled tachyon field with the coupling  $Q = \beta\dot{\rho}_\phi$ . In this case, the eigenvalues of the stationary point in Case (3) are negative. Therefore, it is a stable point.

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